

# Single spin asymmetry in inclusive pion production, Collins effect and the string model (revised version)

**X. Artru**

*Institut de Physique Nucléaire de Lyon, IN2P3-CNRS et Université Claude Bernard,  
F-69622 Villeurbanne Cedex, France*

**J. Czyżewski\***

*Institute of High-Energy Physics, University of Nijmegen,  
Toernooiveld 1, NL-6525 ED Nijmegen, The Netherlands*

**H. Yabuki**

*Department of Mathematics, Hyogo University of Teacher Education,  
Yashiro, Hyogo, 673-14 Japan*

## Abstract

We calculated the single spin asymmetry in the inclusive pion production in the fragmentation region of transversely polarized proton-proton collisions. We generated the asymmetry at the level of fragmentation function (Collins effect) by the Lund coloured string mechanism. We compared our results to the presently available experimental data. We obtained a qualitative agreement with the data after assuming that the transverse polarizations of the  $u$  and the  $d$  quarks in the proton are  $+1$  and  $-1$ , respectively, at  $x_B = 1$ .

**TPJU 13/95**

**hep-ph/9508239**

**August 1995**

---

\* On leave from *Institute of Physics, Jagellonian University, ul. Reymonta 4, PL-30-059 Kraków, Poland*

## 1. Introduction

Quantum Chromodynamics predicts that single transverse spin asymmetries are suppressed in hard collisions, as a consequence of helicity conservation (chiral invariance) in the subprocess. These asymmetries indeed appear as interferences between helicity amplitudes which differ by one unit of helicity, therefore they vanish in the limit  $m_{\text{quark}} \rightarrow 0$ , or equivalently  $Q^2 \rightarrow \infty$  ( $Q$  measures the hardness of the subprocess). Nevertheless, a number of high  $p_T$  reactions persist in showing large asymmetries [1].

These facts do not invalidate QCD but mean that the approach to the asymptotic regime in  $p_\perp$  is very slow, as regards polarization. However, in spite of their “nonasymptotic” character, it is not unreasonable to think that the mechanisms of the asymmetries lie at the parton level. In other words, the asymmetries would be manifestations of quark transverse spin (or transversity). Thus, we could extract information from them about the quark transversity distribution in the nucleon and/or the transversely polarized quark fragmentation. In this paper, we shall present a model for the spin asymmetry in the reaction

$$p \uparrow + p \rightarrow \pi + X \quad (1.1)$$

which, unlike previous approaches [2,3], involves the transverse spin asymmetry of the polarized quark fragmentation [4,5], which hereafter will be referred to as the Collins effect.

The paper is organized as follows: section 2 gives a very short review of the experimental data. In section 3, we explain how the Collins asymmetry can give rise to the observed single spin asymmetry in reaction (1.1) and deduce lower bounds on the transverse polarizations of the quarks in the proton, as well as on the size of the Collins effect. Section 4 presents a quantitative model based on string fragmentation and section 5 gives the numerical results. Section 6 contains discussion of our results and conclusions.

## 2. Main features of single spin asymmetry in inclusive pion production

A strong polarization effect has been observed in recent years by the Fermilab E704 collaboration in the reaction (1.1) with 200 GeV transversely polarized projectile protons [6-8]. The asymmetry is defined as

$$A_N(x_F, p_\perp) \equiv \frac{\sigma \uparrow - \sigma \downarrow}{\sigma \uparrow + \sigma \downarrow}, \quad (2.1)$$

assuming that  $\uparrow$  refers to the  $+\hat{y}$  direction (vertical upwards), and the transverse momentum  $\vec{p}_\perp$  of the pion points towards the  $+\hat{x}$  direction ( $\vec{p}_{\text{beam}}$  is along the  $\hat{z}$  axis). In other words, positive  $A_N$  means that for upward polarization, the pions tend to go to the left.  $x_F = 2p_z^{\text{CM}}/\sqrt{s}$  is the longitudinal momentum fraction of the produced pion and  $p_\perp$  is its transverse momentum.

The data covered two kinematical regions:

- Projectile fragmentation region,  $x_F \geq 0.2$ . In this region the 200GeV E704 data [6-8] show large asymmetries for all pions; positive for  $\pi^+$  and  $\pi^0$  and negative for  $\pi^-$ . The asymmetries vary from about 0 at  $x_F \sim 0.2$  to about +0.4, +0.15 and -0.4, for  $\pi^+$ ,  $\pi^0$  and  $\pi^-$  respectively, at  $x_F \sim 0.7 - 0.8$  and  $p_\perp > 0.7\text{GeV}$ . The earlier 13.3

and 18.5GeV data [9] showed the asymmetry of  $\pi^+$  reaching 0.1 at  $x_F = 0.6$  but that of  $\pi^-$  consistent with zero. However, that measurement was done for  $\pi^+$  and  $\pi^-$  in different  $p_\perp$  regimes.

- Central region,  $x_F \sim 0$ . Ref. [10] reported large positive asymmetry of  $\pi^0$  for the transverse momentum fraction  $x_T = 2p_T/\sqrt{s} > 0.4$ . However, the reanalysis of that data [11] showed the asymmetry consistent with zero in the whole  $x_T$  range covered. Former experiments, at 13.3, 18.5, 24 and 40GeV, [12] observed significant asymmetries in the central region and high  $p_\perp$ .

In this paper we shall concentrate only on the forward fragmentation region.

### 3. Possible explanations of the asymmetry

#### 3.1 Generalities from the parton model.

In the factorized parton model, the cross section for  $p+p \rightarrow \pi+X$  in the forward hemisphere is a convolution of the parton distribution  $q(x, \vec{q}_\perp)$ , the parton-hadron scattering cross section  $\hat{\sigma}_{q+B \rightarrow q'+X} \equiv \hat{\sigma}_{q \rightarrow q'}$  and the parton fragmentation function  $D_{\pi/q'}(z, \vec{h}_\perp)$ . In short-hand notations,

$$\sigma_{A \rightarrow \pi} \approx q \otimes \hat{\sigma}_{q \rightarrow q'} \otimes D_{\pi/q'} \quad (3.1)$$

Each factor in this equation may or may not depend on spin. Transverse polarization can act at three different levels:

- a) in a dependence of  $q(x, \vec{q}_\perp)$  on the azimuth of  $\vec{q}_\perp$  (hereafter referred to as *Sivers effect* [2,13]).
- b) in a single spin asymmetry in  $\hat{\sigma}_{q \rightarrow q'}$  hereafter referred to as *Szwed mechanism* [3]. In this case, (but not necessarily in case a), the quark  $q$  has to inherit a part of the polarization of the proton.
- c) in a dependence of  $D_{\pi/q'}(z, \vec{h}_\perp)$  on the azimuth of  $\vec{h}_\perp$  hereafter referred to as *Collins effect* [4,5]. Here, a transfer of polarization has to occur not only from the proton to quark  $q$  but also from  $q$  to  $q'$ .

#### 3.2 The Collins effect.

According to Collins [4,5], the fragmentation function of the transversely polarized quark  $q$  takes the form

$$D_{\pi/q}(\vec{\mathcal{P}}_q, z, h_\perp) = \bar{D}_{\pi/q}(z, h_\perp) \left\{ 1 + \mathcal{A}_{\pi/q}(z, h_\perp) |\vec{\mathcal{P}}_q| \sin[\varphi(\vec{\mathcal{P}}_q) - \varphi(\vec{h}_\perp)] \right\}, \quad (3.2)$$

where  $\vec{\mathcal{P}}_q$  is the quark polarization vector ( $|\vec{\mathcal{P}}_q| \leq 1$ ),  $\vec{h}_\perp$  is the pion transverse momentum relative to the quark momentum  $\vec{q}$  and  $\varphi(\vec{a})$  is the azimuth of  $\vec{a}$  around  $\vec{q}$ . The factors after  $\mathcal{A}$  can be replaced by  $|\vec{q} \times \vec{h}_\perp|^{-1} \vec{\mathcal{P}}_q \cdot (\vec{q} \times \vec{h}_\perp)$ . Such a dependence is allowed by P- and T- invariance but still remains to be measured.

The Collins effect is the reciprocal of the Sivers effect. However, Collins argued that the latter is prohibited by time reversal invariance [4], while the former is not. As for the mechanism b), it vanishes for massless quarks due to chiral symmetry: single spin

asymmetry in  $q \rightarrow q'$  is not compatible with conservation of quark helicity. Therefore it should be small for hard or semi-hard scattering at high energy. In conclusion, among the sources of asymmetry a), b) and c) discussed above, we have a preference for the Collins effect illustrated by Fig. 1.\*\*

### 3.3 Consequence for the single spin asymmetry.

Let us consider the hypothesis that the E704 asymmetry is due to the Collins effect. In the parton model, the polarized inclusive cross section reads

$$\frac{d\sigma}{d^3\vec{p}} = \sum_{a,b,c,d} \int dx q_a(x) \int dy q_b(y) \times \int d\cos\hat{\theta} d\hat{\varphi} \frac{d\hat{\sigma}^{q_a+q_b \rightarrow q_c+q_d}}{d\hat{\Omega}} \int dz d^2h_{\perp} D_{\pi/q_c}(\vec{P}_{q_c}, z, \vec{h}_{\perp}) \delta(\vec{p} - z\vec{c} - \vec{h}_{\perp}), \quad (3.3)$$

where  $\vec{c}$  is the momentum of the scattered quark. The transverse polarization of that quark is given by

$$\vec{P}_{q_c} = \mathcal{R} \vec{P}_{\text{beam}} \frac{\Delta_{\perp} q_a(x)}{q_a(x)} \hat{D}_{NN}(\hat{\theta}). \quad (3.4)$$

$\mathcal{R}$  is the rotation about  $\vec{p}_{\text{beam}} \times \vec{c}$  which brings  $\vec{p}_{\text{beam}}$  along  $\vec{c}$ ,

$$\Delta_{\perp} q(x) \equiv q \uparrow(x) - q \downarrow(x), \quad (3.5)$$

also called  $h_1(x)$ , is the quark transversity distribution [15,16] in the proton polarized upwards, and  $\hat{D}_{NN}(\hat{\theta})$  is the coefficient of the spin transfer, normal to the scattering plane, in the subprocess. Formula (3.1) results from integration of (3.3) over the momentum fraction  $y$  of the parton of the target.  $q_b$  and  $q_d$  in (3.3) may also be replaced by gluons. This does not change the results concerning the asymmetry.

At large  $x_F$ , the dominant quark flavours are  $q_a = q_c = u$  for  $\pi^+$  production and  $q_a = q_c = d$  for  $\pi^-$  production. Furthermore, the hard scattering occurs predominantly at small  $\hat{\theta}$  and  $\hat{D}_{NN}(\hat{\theta})$  is close to unity, as in the case of the  $\hat{t}$ -channel one-gluon exchange where  $\hat{D}_{NN} = -2\hat{s}\hat{u}/(\hat{s}^2 + \hat{u}^2)$ ;  $\hat{s}$  and  $\hat{u}$  being the Mandelstam variables of the parton subprocess. Thus, the results of E704 collaboration imply

$$\frac{\Delta_{\perp} u(\bar{x})}{u(\bar{x})} \mathcal{A}(\bar{z}, \bar{h}_{\perp}) \geq \text{about } 0.4, \quad (3.6)$$

$$\frac{\Delta_{\perp} d(\bar{x})}{d(\bar{x})} \mathcal{A}(\bar{z}, \bar{h}_{\perp}) \leq \text{about } -0.4, \quad (3.7)$$

for  $\bar{x}\bar{z} \simeq x_F \simeq 0.8$ .  $\bar{x}$  means the most probable value of  $x$ . The inequalities take into account the fact that integration over  $\vec{q}_{\perp}$  and  $\hat{\theta}$  always dilutes the Collins asymmetry. It

---

\*\* We shall not discuss other approaches [14] not relying on the factorized parton description (Eq. (3.1) or (3.3)). They are not necessarily in contradiction with the present one.

means that we get at least a lower bound of 0.4 separately for  $|\Delta_\perp u/u|$ ,  $|\Delta_\perp d/d|$  at large  $x$  and  $|\mathcal{A}(z, h_\perp)|$  at large  $z$  and for the most probable value of  $h_\perp$ .

#### 4. Simple model of single-spin asymmetry

In order to make the conclusions of the previous section more quantitative, we have constructed a simple model based on the string model [17] of quark fragmentation.

We consider only the valence quarks of the projectile proton. We assume that the quark elastic-scattering cross-section  $d\hat{\sigma}/dq_\perp$  in (3.3) depends only on the transverse momentum  $q_\perp$  of the scattered quark. Since the scattering angle is in our case very small we assume  $D_{NN} = 1$  in Eq. (3.4). After scattering, the quark spans a string between itself and the target. The string decays according to the recursive Lund recipe [17], for which we use the Standard Lund splitting function:

$$f(z) = (1 + C)(1 - z)^C, \quad (4.1)$$

$z$  being the fraction of the null plane momentum  $p^+ \equiv p^0 + p^3$  of the string taken away by the next hadron.  $f(z) = D^{\text{rank}=1}(z)$  corresponds to the production rate of the leading\*\*\* hadron. This gives for all ranks the fragmentation function [17]:

$$D(z) = (1 + C)\frac{1}{z}(1 - z)^C = \frac{1}{z}f(z). \quad (4.2)$$

Thus, for all the other (subleading) hadrons originating from the string we get:

$$D^{\text{rank} \geq 2}(z) = (1 + C)\frac{1 - z}{z}(1 - z)^C = f(z)\frac{1 - z}{z}. \quad (4.3)$$

The transverse momenta, relative to the direction of the string, of the quark ( $\vec{k}_{q\perp}$ ) and the antiquark ( $\vec{k}_{\bar{q}\perp}$ ) of every pair created in the string balance each other (local compensation of the transverse momentum) and are distributed according to

$$\rho(\vec{k}_\perp) d^2\vec{k}_\perp = \frac{d^2\vec{k}_\perp}{\kappa} \exp\left(\frac{-\pi k_\perp^2}{\kappa}\right), \quad (4.4)$$

$\kappa$  being the string tension.

Each quark-antiquark pair created during the string breaking is assumed to be in a  $^3P_0$  state (vacuum quantum numbers) [18], *i.e.*, with parallel polarizations. According to the Lund mechanism for inclusive  $\Lambda$  polarization [17], their polarizations are correlated to the transverse momentum  $k_{\bar{q}\perp}$  of the antiquark by

$$\vec{\mathcal{P}}_{\bar{q}}(\vec{k}_{\bar{q}\perp}) = -\frac{L}{\beta + L} \frac{\hat{z} \times \vec{k}_{\bar{q}\perp}}{k_{\bar{q}\perp}}, \quad (4.5)$$

---

\*\*\* We call "leading" or "first-rank" the hadron which contains the original quark spanning the string.

where  $\hat{z}$  is the unit vector along the  $z$  direction,  $\beta$  is the parameter determining the correlation and  $L$  is the classical orbital angular momentum of the  $q\bar{q}$  pair:

$$L = \frac{2 k_{\bar{q}\perp} \sqrt{m_q^2 + k_{\bar{q}\perp}^2}}{\kappa} \simeq \frac{2 k_{\bar{q}\perp}^2}{\kappa} \quad (4.6)$$

(see Fig. 2 for the schematic explanation).

In order that  $q_0$  and  $\bar{q}_1$  of Fig. 2 combine into a pion, they have to form a spin singlet state, the probability of which is

$$\frac{1}{4} (1 - \vec{\mathcal{P}}_{q_0} \cdot \vec{\mathcal{P}}_{\bar{q}_1}), \quad (4.7)$$

in accordance with the projection operator on the singlet state  $\frac{1}{4} - \vec{s}(q_0) \cdot \vec{s}(\bar{q}_1)$ ,  $\vec{s}$  being the quark spin operator. The polarization of the leading quark  $q_0$  is

$$\vec{\mathcal{P}}_{q_0}(x) = \frac{\Delta_{\perp} q_0(x)}{q_0(x)} \cdot \hat{y}. \quad (4.8)$$

The factor (4.7) causes the Collins effect: if  $q_0$  in Fig. 2 is polarized upwards then  $\bar{q}_1$  — and the pion which contains  $\bar{q}_1$  — tends to go to the left-hand-side of the  $\hat{z}$  direction.

Putting together Eqs (4.1), (4.4) and (4.7), we get the contribution of the leading pions to the fragmentation function of the polarized quark  $q$ :

$$D_{\pi/q}^{\text{rank}=1}(z, \vec{h}_{\perp}) = c_1 D^{\text{rank}=1}(z) \rho(h_{\perp}) (1 - \vec{\mathcal{P}}_q \cdot \vec{\mathcal{P}}_{\bar{q}}(\vec{h}_{\perp})), \quad (4.9)$$

where  $\vec{\mathcal{P}}_{\bar{q}}$  is given by (4.5) and (4.6).  $c_1$  is the probability that the flavours of  $q$  and  $\bar{q}$  combine into the pion of the appropriate charge. We do not take into account the vector mesons since at high  $x_F$  the pions are mostly produced directly. Therefore we omit the factor  $\frac{1}{4}$  of Eq. (4.7).

We do not introduce the Collins effect in subleading ranks. The subleading fragmentation function is spin independent and reads:

$$D_{\pi/q}^{\text{rank} \geq 2}(z, \vec{h}_{\perp}) = c_2 D^{\text{rank} \geq 2}(z) \rho_{\pi}(h_{\perp}), \quad (4.10)$$

where  $\rho_{\pi}(h_{\perp})$  is the distribution of the transverse momentum of the produced pion with respect to the direction of the fragmenting quark. It is the convolution of the transverse momentum distributions of its constituents (4.4) and is also a Gaussian function but of the twice larger variance.  $c_2$  is the flavour factor analogical to that of Eq. (4.9).

We do not take into account the second string which is spanned by the remnant diquark of the projectile. A large part of the energy of that string goes into the leading baryon and it does not contribute much to the pion spectrum at high  $x_F$ .

Our final formula for the polarized cross-section reads:

$$\frac{d\sigma}{dx_F d^2\vec{p}_\perp} = \sum_{q=u,d} \int dx q(x) \int d^2\vec{q}_\perp \frac{d\hat{\sigma}}{d^2\vec{q}_\perp} \int dz d^2\vec{h}_\perp D_{\pi/q}(z, \vec{h}_\perp) \times$$

$$\delta \left( x_F - \sqrt{z^2 x^2 - \frac{4p_\perp^2}{s}} \right) \delta^2(\vec{p}_\perp - z\vec{q}_\perp - \vec{h}_\perp), \quad (4.11)$$

where

$$D_{\pi/q}(z, \vec{h}_\perp) = D_{\pi/q}^{\text{rank}=1}(z, \vec{h}_\perp) + D_{\pi/q}^{\text{rank} \geq 2}(z, \vec{h}_\perp). \quad (4.12)$$

Since the production rate varies with the azimuthal angle  $\phi$  of the pion momentum like in (3.2) then, in order to obtain the asymmetry at given values of  $p_\perp$  and  $x_F$ , we need to compare  $d\sigma(x_F, p_\perp, \phi)$  only at  $\phi = 0$  and  $\phi = \pi$ . Thus, for the transverse spin asymmetry we get:

$$A_N(x_F, p_\perp) = \frac{d\sigma(x_F, p_\perp, 0) - d\sigma(x_F, p_\perp, \pi)}{d\sigma(x_F, p_\perp, 0) + d\sigma(x_F, p_\perp, \pi)} \quad (4.13)$$

.

## 5. Numerical results

For the numerical calculations we parametrized the quark distributions as follows:

$$u(x) = \frac{16}{3\pi} x^{-1/2} (1-x)^{3/2}$$

$$d(x) = \frac{15}{16} x^{-1/2} (1-x)^2. \quad (5.1)$$

One has to note that the scale of the process we are dealing with here is usually below 1 GeV. This means that one cannot use the quark distributions obtained from the deeply inelastic scattering at high  $Q^2$ . In this region we have to use a parametrization being between the large  $Q^2$  region, where, at high  $x$ ,  $u(x) \sim (1-x)^3$  and  $d(x) \sim (1-x)^4$  and the dual parton model region, where the quark-diquark splitting function,  $q(x) \sim (1-x)$ .

We have used the string tension  $\kappa = 0.197 \text{ GeV}^2$  (it corresponds to 1 GeV/fm) and the parameter of the fragmentation function  $C = 0.5$ . In pair creation we have used the flavour abundances with the ratio  $u : d : s = 3 : 3 : 1$ , which determines the coefficients in Eqs (4.9) and (4.10) to be  $c_1 = 3/7$ ,  $c_2 = 9/49$  for charged and  $c_2 = 18/49$  for neutral pions<sup>§</sup>.

---

<sup>§</sup> In this model, every  $u\bar{u}$  or  $d\bar{d}$  meson is considered as a  $\pi^0$  (no  $\eta^0$ ) ; it gives  $\sigma(\pi^+) + \sigma(\pi^-) = \sigma(\pi^0)$ , instead of  $2 \sigma(\pi^0)$  as required by isospin. Nevertheless, Eq. (5.8) below remains true.

The quark scattering cross-section was parametrized as

$$\frac{d\sigma}{d^2q_\perp} \sim \frac{1}{(q_\perp^2 + M^2)^3} \quad (5.2)$$

with the parameter  $M^2 = 0.5\text{GeV}^2$  making this cross-section normalizable.

Such a parametrization, with the power-law decrease advocated by Field and Feynman [19], gives, up to the overall normalization, a good agreement with the experimental inclusive spectra of pions. We show the comparison to the 400GeV and 360GeV experimental data [20,21] in Fig. 3.

As it was noted in section 2, if one assumes the Collins effect to be responsible for the observed asymmetry, then the E704 data implies  $\Delta_\perp u/u = -\Delta_\perp d/d$  at high  $x$  thus indicating a violation of  $SU(6)$ , where  $\Delta_\perp u/u = 2/3$  and  $\Delta_\perp d/d = -1/3$ . For the numerical calculations of the single spin asymmetry, we assumed that

$$\Delta_\perp u/u = -\Delta_\perp d/d = \mathcal{P}(x) = \mathcal{P}_{\max} x^n. \quad (5.3)$$

We found that  $\mathcal{P}_{\max} = 1$  and  $n = 2$  gives the best agreement with the experimental  $x_F$  dependence of the asymmetry.

Such a transversity does not violate the positivity constraints derived recently by Soffer [22]. The Soffer's inequality relates the transversity distribution  $\Delta_\perp q = h_1$  to the helicity distribution  $\Delta q = g_1$  and reads:

$$2|\Delta_\perp q| \leq q + \Delta q. \quad (5.4)$$

If one takes into account only the valence quarks, then the helicity asymmetries of the proton, measured in the deeply inelastic scattering, is:

$$A_1^p = \frac{4\Delta u + \Delta d}{4u + d} \quad (5.5)$$

and that of the neutron equals:

$$A_1^n = \frac{\Delta u + 4\Delta d}{u + 4d}. \quad (5.6)$$

Thus, if we assume that  $\Delta_\perp u/u = -\Delta_\perp d/d = \mathcal{P}(x)$ , then the Soffer's inequality implies

$$\begin{aligned} A_1^p(x) &\geq 2\mathcal{P}(x) - 1, \\ A_1^n(x) &\geq 2\mathcal{P}(x) - 1. \end{aligned} \quad (5.7)$$

The above inequalities are satisfied by the present data if one assumes  $\mathcal{P}(x) = x^2$ . This is shown in Fig. 4 where  $A_1^p(x)$  measured by SMC [23] and E143 [24] and  $A_1^n(x)$  measured by the E142 collaboration [25] are plotted together with the curves representing Eqs (5.7). One can see that  $\mathcal{P}(x) = x^2$  is well within the limits and  $\mathcal{P}(x) = x$  is still allowed by the data, as well as the powers of  $x$  higher than 2.



We plot the results of our calculation of the transverse spin asymmetry in Figs. 5–8. The full lines and the dashed ones in Figs 7 and 8 show the predictions of our model obtained with the parameter  $\beta$  of Eq. (4.5) equal 1 [17].

These predictions are in reasonable agreement with most of the data. Only the asymmetry measured by the E704 collaboration at  $0.7\text{GeV} < p_\perp < 2\text{GeV}$  (Fig. 5a) is strongly underestimated. Our model gives the opposite asymmetries for  $\pi^+$  and  $\pi^-$  and predicts the increase of the absolute values of the asymmetries with  $x_F$ . Nevertheless, it cannot account for the very strong  $p_\perp$  dependence of the asymmetries measured by E704. We got agreement with the low- $p_\perp$  data but underestimate the high- $p_\perp$  ones.

However, the  $p_\perp$  dependence of the E704 data was discussed recently by Arestov [26] from the purely experimental point of view and was found to be questionable. Also the asymmetries of  $\pi^0$ , measured by E704 in the central region, showed initially a very strong  $p_\perp$  dependence [10] but after reanalysis [11] showed no such dependence at all and are consistent with zero. Both the lack of the strong  $p_\perp$  dependence of the asymmetry and its very small magnitude in the central region are in agreement with the Collins effect.

In the previous version of this paper [27] we were able to obtain the strong  $p_\perp$  dependence of the asymmetry at high  $x_F$  but it was only due to the fact that we neglected the quark scattering and assumed only the exponentially falling intrinsic  $q_\perp$  distribution of the leading quark, similar to the  $h_\perp$  distribution in the fragmentation. In the present approach, the  $q_\perp$  distribution (5.2) in quark scattering is much flatter than the  $h_\perp$  distribution (4.4). Thus, at high  $p_\perp$  the contribution of the transverse momentum of the fragmentation (which determines the asymmetry) to the total  $p_\perp$  saturates. This makes the asymmetries rise at rather low  $p_\perp$  and then flatten at higher  $p_\perp$ .

We checked, by forcing  $\beta$  in (4.5) to be 0, that the strong enough Collins effect (satisfying the inequalities (3.6) and (3.7)) can account for the magnitude of the experimental asymmetries of Fig. 5a. Nevertheless,  $\beta = 0$  (100% spin- $k_\perp$  correlation in string breaking independent of  $k_\perp$ ) is not thinkable. It does not change the  $p_\perp$  dependence either; so large Collins effect leads to strong overestimation of the lower- $p_\perp$  data.

As regards the region where both  $x_F$  and  $p_\perp$  are low (small  $x_F$  points of Fig. 5b), the calculated asymmetry can be slightly overestimated. At so low  $p_\perp$  and rather small  $x_F$ , the pions produced in the decay of the second string, spanned by the diquark, can contribute significantly and wash out the asymmetry.

The asymmetry of  $\pi^0$  is a combination of the  $\pi^+$  and  $\pi^-$  ones:

$$A_N(\pi^0) = \frac{\sigma(\pi^+) A_N(\pi^+) + \sigma(\pi^-) A_N(\pi^-)}{\sigma(\pi^+) + \sigma(\pi^-)}. \quad (5.8)$$

This relation follows from the isospin symmetry for isoscalar targets. For the proton target it holds provided that the isospin correlations are of short range in rapidity; this is the case in the multiparticle production. Nevertheless, we show in Fig. 6 the comparison of our results to the E704 data [7,8] on  $\pi^0$  production in  $pp$  and  $\bar{p}p$  collisions. Here there is no discrepancy between the model and the data. Note that our model predicts the same asymmetry for the proton and the antiproton beams.

For completeness we show in Fig. 7 the earlier data measured with 13.3 and 18.5 GeV polarized protons [9]. Here the agreement of the model and the data is also good. Only

the asymmetry of  $\pi^-$  calculated in the model tend to overestimate the data, particularly at low  $x_F$ . However, the  $\pi^-$  data has been measured at very low  $p_\perp$  and the contribution of the diquark fragmentation can be not negligible also here.

Finally, in Fig. 8 we show the predictions of the model for the asymmetry of charged kaons. Since in this model the asymmetry is a purely leading effect, the asymmetry of  $K^-$  vanishes. Measuring this asymmetry and its  $x_F$  dependence would provide information on whether and to what extent the asymmetry is limited to the leading particle. From the point of view of our model, the nonvanishing asymmetry of  $K^-$  would be an indication of the Collins effect in higher-rank hadrons.

The asymmetry of  $K^+$  is predicted to be similar to that of the positive pions.

## 6. Discussion and conclusions

To summarize, we have calculated the single transverse spin asymmetry in high-energy  $pp$  collisions in a simple model involving the Collins effect (asymmetry arising at the level of fragmentation of a quark into hadrons). We parametrized the Collins effect by the Lund mechanism of polarization in the coloured string model.

We got qualitative agreement with the data when we assumed that:

- a) The transverse polarization of the  $u$  and  $d$  quarks in the transversely polarized proton are close to unity but of the opposite sign ( $\vec{\mathcal{P}}_u = \vec{\mathcal{P}}_{\text{proton}}$ ,  $\vec{\mathcal{P}}_d = -\vec{\mathcal{P}}_{\text{proton}}$ ) at momentum fraction  $x$  close to 1.
- b) The dependence of the quark transversity (or polarization) on the momentum fraction  $x$  is close to be proportional to  $x^2$ .

However, the Collins effect cannot explain the very strong  $p_\perp$  dependence of the E704 data. Some additional mechanism of the asymmetry would be needed in order to account for the E704 data for charged pions at  $p_\perp > 0.7\text{GeV}$ . Apart of this set, our model gives good agreement with the data. Presently, we do not find any mechanism which could remove the above discrepancy.

The quark transversities we inferred at large  $x$ :

$$\frac{\Delta_\perp u(x)}{u(x)} \approx -\frac{\Delta_\perp d(x)}{d(x)} \rightarrow 1 \quad (\text{for } x \rightarrow 1) \quad (6.1)$$

are, in fact, not unreasonably large. Consider a covariant model of the baryon consisting of a quark and a bound spectator diquark [16,28] ; then

$$q \uparrow (x) = \frac{x}{16\pi^2} \int_{-\infty}^{q_m^2} dq^2 \left( \frac{g(q^2)}{q^2 - m_q^2} \right)^2 \sum_{\text{diquark polarization}} |\bar{u}(q \uparrow) V u(p \uparrow)|^2 \quad (6.2)$$

where  $g(q^2)$  is the  $q - q\bar{q} - B$  form factor,  $V = 1$  for a scalar diquark,  $V = \gamma_5 \gamma \cdot \varepsilon$  for a  $1^+$  diquark of polarization  $\varepsilon^\mu$  and

$$q_m^2 = x m_B^2 - \frac{x}{1-x} m_{q\bar{q}}^2. \quad (6.3)$$

Formula (6.2) is similar to the covariant Weizsäcker–Williams formula, but for a “spin  $\frac{1}{2}$  cloud”). Independently of  $g(q^2)$ , this model predicts the following behaviour at  $x \rightarrow 1$ :

- for a  $1^+$  spectator diquark, helicity is fully transmitted ( $\Delta_L q(x)/q(x) \rightarrow 1$ ), transversity is fully reversed ( $\Delta_\perp q(x)/q(x) \rightarrow -1$ ). In particular,  $\mathcal{P}_d(x) \rightarrow -1$ .
- for a  $0^+$  diquark,  $\Delta_\perp q(x)$  and  $q^+(x)$  coincide and, for  $g(q^2)$  decreasing faster than  $q^{-2}$ , they exceed  $\frac{2}{3}q(x)$  as  $x \rightarrow 1$

Thus, a dominance of the scalar spectator for the  $u$  quark and the pseudo-vector one for the  $d$  at  $x \sim 1$  could lead to the large opposite transversities as in (6.1).

The conclusion b), related to the  $x_F$  dependence, is model-dependent and does not need to be considered as a firm prediction. One needs a good parametrization of the Collins effect before one can deduce the  $x$  dependence of the quark transversity. Our parametrization is an approximation which should work only at reasonably high values of  $x_F$ . We took into account the Collins effect only for the first-rank (leading) hadrons, wherefrom  $\mathcal{A} \propto z$  in (3.2). In the string model, the second-rank hadrons have the asymmetry of the opposite sign as compared to the first-rank ones. More generally, the subsequent ranks are asymmetric in the opposite way to each other (as required also by local compensation of transverse momentum). This should cause a faster decrease of  $\mathcal{A}$  at lower  $z$  values, where the higher-rank hadrons are more important. Unfortunately this feature was not possible to include in our simple semi-analytical calculation, since the yields of rank-2 (and higher) hadrons do not have simple analytical forms. Assuming  $\bar{x} \sim \bar{z} \sim \sqrt{x_F}$ , a steeper  $\mathcal{A}$  (for instance  $\propto z^2$ ) would have to be compensated by a flatter  $\mathcal{P}_q(x)$  (for instance  $\propto x$ ).

The contribution of vector mesons also should reduce  $\mathcal{A}$  at lower  $z$ . Moreover, it has been shown that the vector meson can also have a *tensor* polarization [29] which would result in the Collins effect for the decay products. We did not include this possibility. Another mechanism of asymmetry can be the interference between direct and resonance production [30].

The main conclusion of this paper is that the single spin asymmetry may be the first experimental indication for the existence of the Collins effect. A more detailed experiment would be useful to select between this and alternative explanations. Besides its theoretical interest, the Collins effect may be the most efficient “quark polarimeter” necessary for the measurements of the transversity distributions in the nucleons [5,31]. We hope that this effect will soon be tested directly, for instance in the azimuthal correlation of two pion pairs from opposite quark jets in  $e^+e^-$  annihilation.

## Acknowledgements

We are grateful to J. Szwed for discussions. X.A. and J.C. acknowledge the financial support from the IN2P3–Poland scientific exchange programme. J.C. has been also supported by the Polish Government grants of KBN no. 2 0054 91 01, 2 0092 91 01 and 2 2376 91 02 during completion of this work.

## References

- [1] K. Heller, 7<sup>th</sup> Int. Conf. on Polarization Phenomena in Nuclear Physics, Paris 1990, p. 163, and references therein; P.R. Cameron *et al.*, Phys. Rev. **D32**, 3070 (1985); T.A. Armstrong *et al.*, Nucl. Phys. **B262**, 356 (1985); S. Gourlay *et al.*, Phys. Rev. Lett. **56**, 2244 (1986); M. Guanziroli *et al.*, Z. Phys. **C37**, 545 (1988)
- [2] D. Sivers, Phys. Rev. **D41**, 83 (1990); Phys. Rev. **D43**, 261 (1991)
- [3] J. Szwed, Proc. of the 9<sup>th</sup> International Symposium “High Energy Spin Physics” held at Bonn, 6–15 Sep. 1990, Springer Verlag 1991; Phys. Lett. **B105**, 403 (1981)
- [4] J. Collins, Nucl. Phys. **B396**, 161 (1993)
- [5] J. Collins, S.F. Heppelmann and G.A. Ladinsky, Nucl. Phys. **B420**, 565 (1994)
- [6] E704 Coll., D.L. Adams *et al.*, Phys. Lett. **B264**, 462 (1991)
- [7] E704 Coll., D.L. Adams *et al.*, Z. Phys. **C56**, 181 (1992); E704 Coll., B.E. Bonner *et al.*, Phys. Rev. Lett. **61**, 1918 (1988)
- [8] E704 Coll., D.L. Adams *et al.*, Phys. Lett. **B261**, 201 (1991)
- [9] B.E. Bonner *et al.*, Phys. Rev. **D41**, 13 (1990);
- [10] E704 Coll., D.L. Adams *et al.*, Phys. Lett. **B276**, 531 (1992)
- [11] E704 Coll., D.L. Adams *et al.*, Protvino preprint IHEP-94-88, IFVE-94-88 (1994)
- [12] V.D. Apokin *et al.*, Phys. Lett. **B243**, 461 (1990); M.S. Amaglobeli *et al.*, Sov. J. Nucl. Phys. **50**, 432 (1989); J. Antille *et al.*, Phys. Lett. **B94**, 523 (1980); S. Saroff *et al.*, Phys. Rev. Lett. **64**, 995 (1990)
- [13] M. Anselmino, M. Boglione, F. Murgia, INFN Cagliari preprints DFTT-47-94 and DFTT-48-94 (1994)
- [14] M.G. Ryskin, Sov. J. Nucl. Phys. **48**, 708 (1988); C. Boros, Liang Zuo-tang and Meng Ta-chung, Phys. Rev. Lett. **70**, 1751 (1993); H. Fritzsche, Mod. Phys. Lett. **A5**, 625 (1990); P.G. Ratcliffe, Proc. 10th International Symposium on High Energy Spin Physics (Yamada Conference XXXV), November 9-14, 1992, Nagoya, Japan (edited by T. Hasegawa, N. Norikawa, A. Masaike, S. Sawada), Universal Academy Press, Inc. p. 635
- [15] J.P. Ralston and D.E. Soper, Nucl. Phys. **B152**, 109 (1979); J.L. Cortes, B. Pire and J.P. Ralston, Z. Phys. **C55**, 409 (1992); R.L. Jaffe and Xiangdong Ji, Nucl. Phys. **B375**, 527 (1992)
- [16] X. Artru and M. Mekhfi, Z. Phys. **C45**, 669 (1990)
- [17] B. Andersson, G. Gustafson, G. Ingelman, T. Sjöstrand, Phys. Rep. **97** 31 (1983)
- [18] A. Le Yaouanc, L. Oliver, O. Pène and J.-C. Raynal, *Hadron Transitions in the Quark Model* (Gordon and Breach, 1988)
- [19] R.D. Field, R.P. Feynman, Phys. Rev. **D15**, 2590 (1977)
- [20] NA27 Coll., M. Aguilar-Benitez *et al.*, Z. Phys. **C50**, 405 (1991)
- [21] NA23 Coll., J.L. Bailly *et al.*, Z. Phys. **C35**, 309 (1987)
- [22] J. Soffer, Phys. Rev. Lett. **74**, 1292 (1995)
- [23] SMC Coll., D. Adams *et al.*, Phys. Lett. **B329**, 399 (1994)
- [24] E143 Coll., K. Abe *et al.*, Phys. Rev. Lett. **74**, 346 (1995)
- [25] E142 Coll., D.L. Anthony *et al.*, Phys. Rev. Lett. **71**, 959 (1993)
- [26] Yu.I. Arestov, Phys. Lett. **B333**, 255 (1994)

- [27] X. Artru, J. Czyżewski, H. Yabuki, preprint **TPJU-12/94** (Cracow, 1994) and **LYCEN/9423** (Lyon), hep-ph/9405426, unpublished
- [28] H. Meyer and P.J. Mulders, Nucl. Phys. **A528**, 589 (1991)
- [29] Xiangdong Ji, Phys. Rev. **D49**, 114 (1994)
- [30] J.C. Collins and G.A. Ladinsky, PSU/TH/**114** (hep-ph/9411444)
- [31] X. Artru, QCD and High Energy Hadronic Interactions (Ed. J. Trân Thanh Vân, Editions Frontières, 1993), p. 47

## Figure captions

- Fig. 1 Inclusive pion production. Two events (a) and (b), symmetric with respect to the  $\hat{y}\hat{z}$  plane, are represented. Without polarization, they would have the same probability. In the polarized case, the Collins effect favours the case (a). The arrows labelled  $q_i$  represent the momenta of the quarks in the subprocess. The spins are denoted by the arch-like arrows. The Collins effect acts at the last stage, where the quark  $q_c$  fragments into the pion carrying momentum  $\vec{p}$ .  $h_\perp$  is the pion's transverse momentum with respect to the quark  $q_c$ .
- Fig. 2 Production of the leading pion in a string spanned by the transversely spinning quark  $q_0$ .
- Fig. 3 Inclusive cross-section for production of  $\pi^+$  and  $\pi^-$  plotted versus  $x_F$  and  $p_\perp^2$ . The 360GeV data come from [21] and the 400GeV data from [20]. The dashed curve corresponding to  $\pi^+$  and the solid one corresponding to  $\pi^-$  show the results of Eq. (4.11) together with (5.1) and (5.2).
- Fig. 4 Longitudinal spin asymmetries of the proton and of the neutron measured in deeply inelastic scattering. The full squares are the E143 data [24], the full circles the SMC data [23] and the open squares represent the E142 data [25]. The values of  $A_1^p$  and  $A_1^n$  allowed by the Soffer's inequality (5.4) if  $|\Delta_\perp q|/q = x^2$  are denoted by the hatched area. One can see that  $|\Delta_\perp q|/q = x$  is also allowed by the present data.
- Fig. 5 Single spin asymmetry measured by E704 collaboration for charged pions at  $0.2 < p_\perp < 2.0\text{GeV}$  [6]. The curves are our model results calculated with quark transverse polarizations  $\Delta_\perp u/u = -\Delta_\perp d/d = x^2$  and  $\beta = 1$ .
- Fig. 6 The asymmetry of  $\pi^0$ 's produced in  $pp$  and  $\bar{p}p$  collisions measured by the E704 collaboration [7,8]. The curve is our model prediction.
- Fig. 7 The single spin asymmetries of charged pions measured with 13.3 and 18.5GeV proton beams [9]. The dashed lines are the predictions of our model for 13.3GeV and the full ones are for 18.5GeV.
- Fig. 8 Transverse spin asymmetry of the charged kaons as predicted by our model. No asymmetry is predicted for  $K^-$  when the spin effects are assumed to act only on the leading hadron.

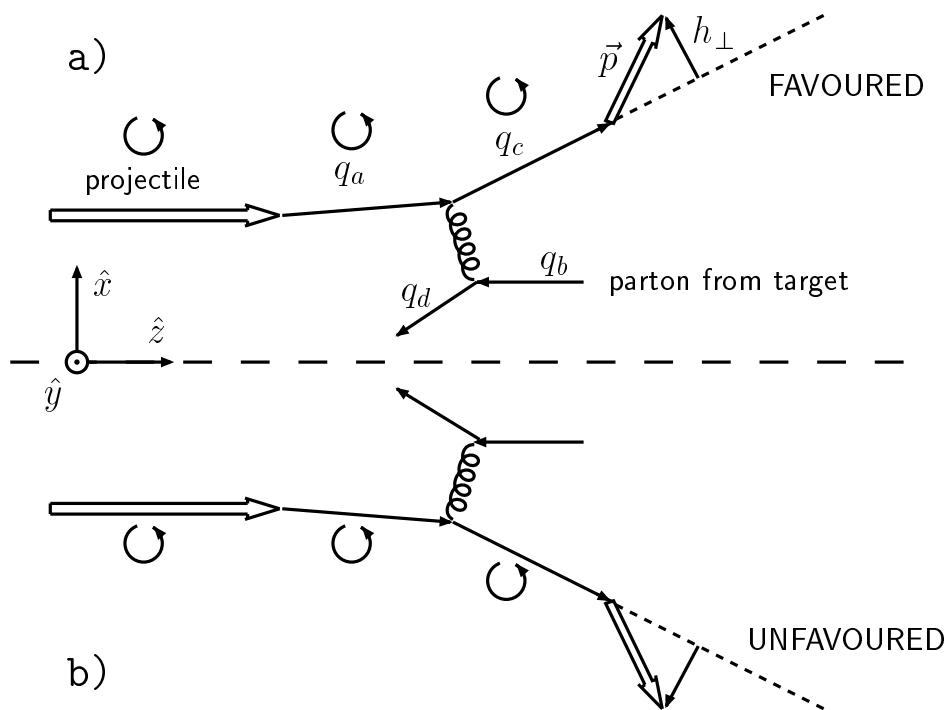


Fig. 1

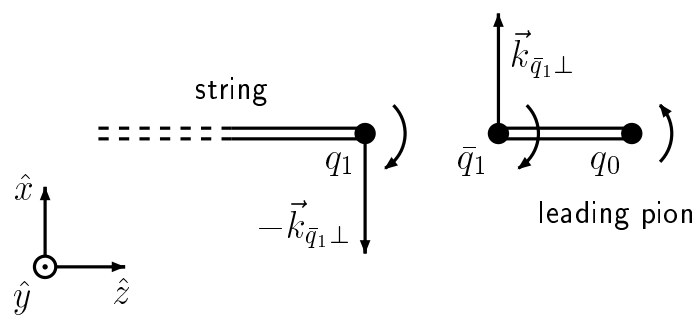


Fig. 2



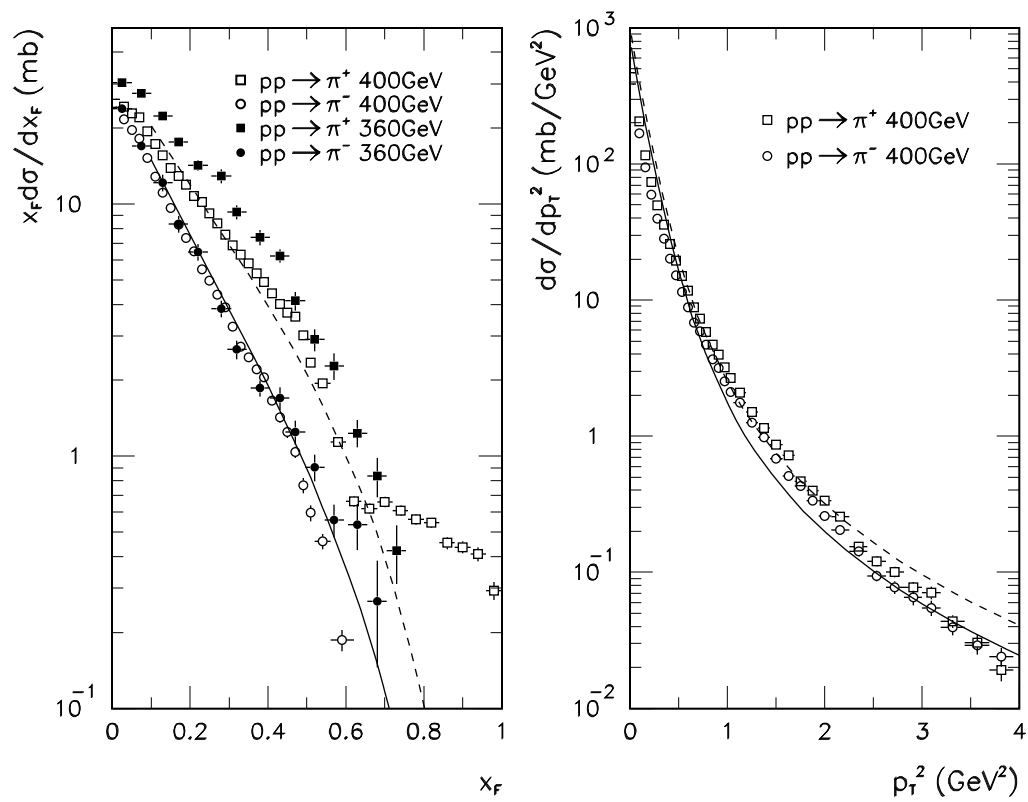


Fig.3

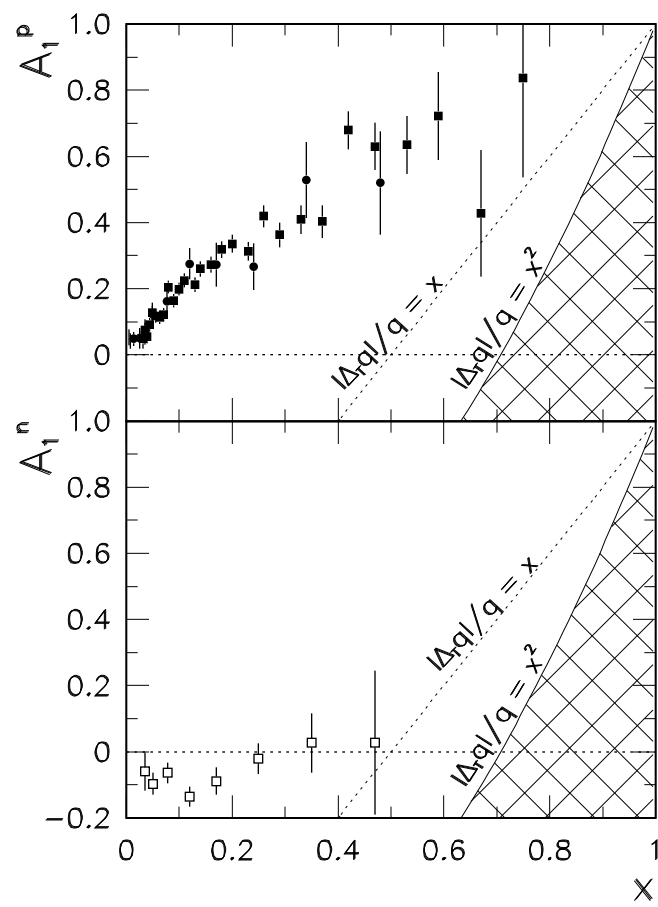


Fig.4

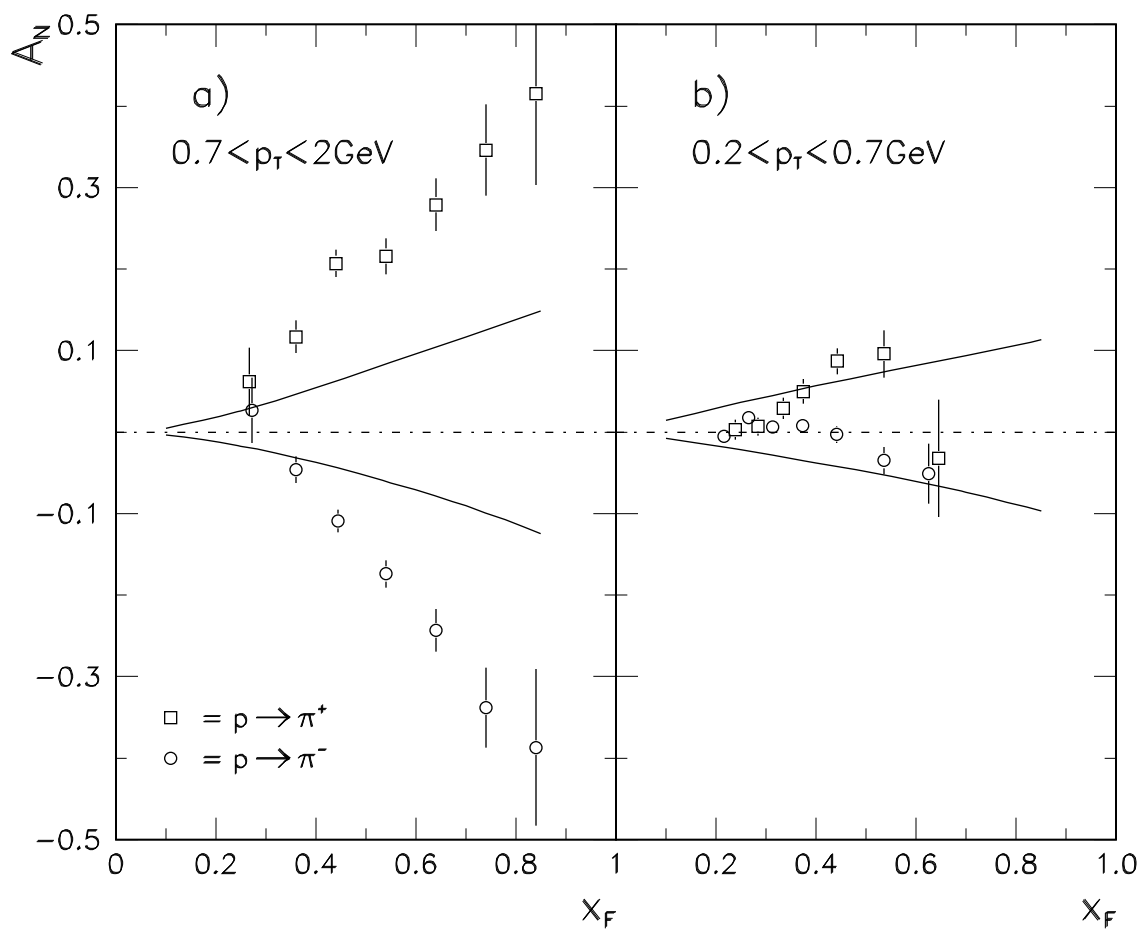


Fig. 5

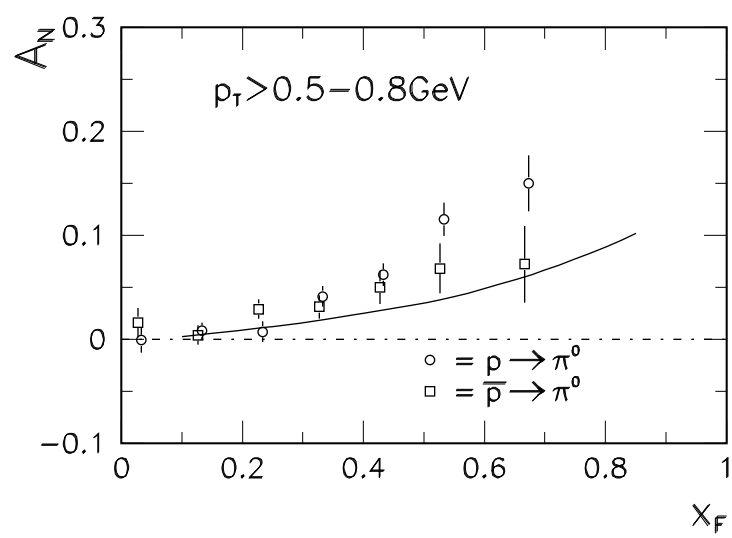


Fig. 6

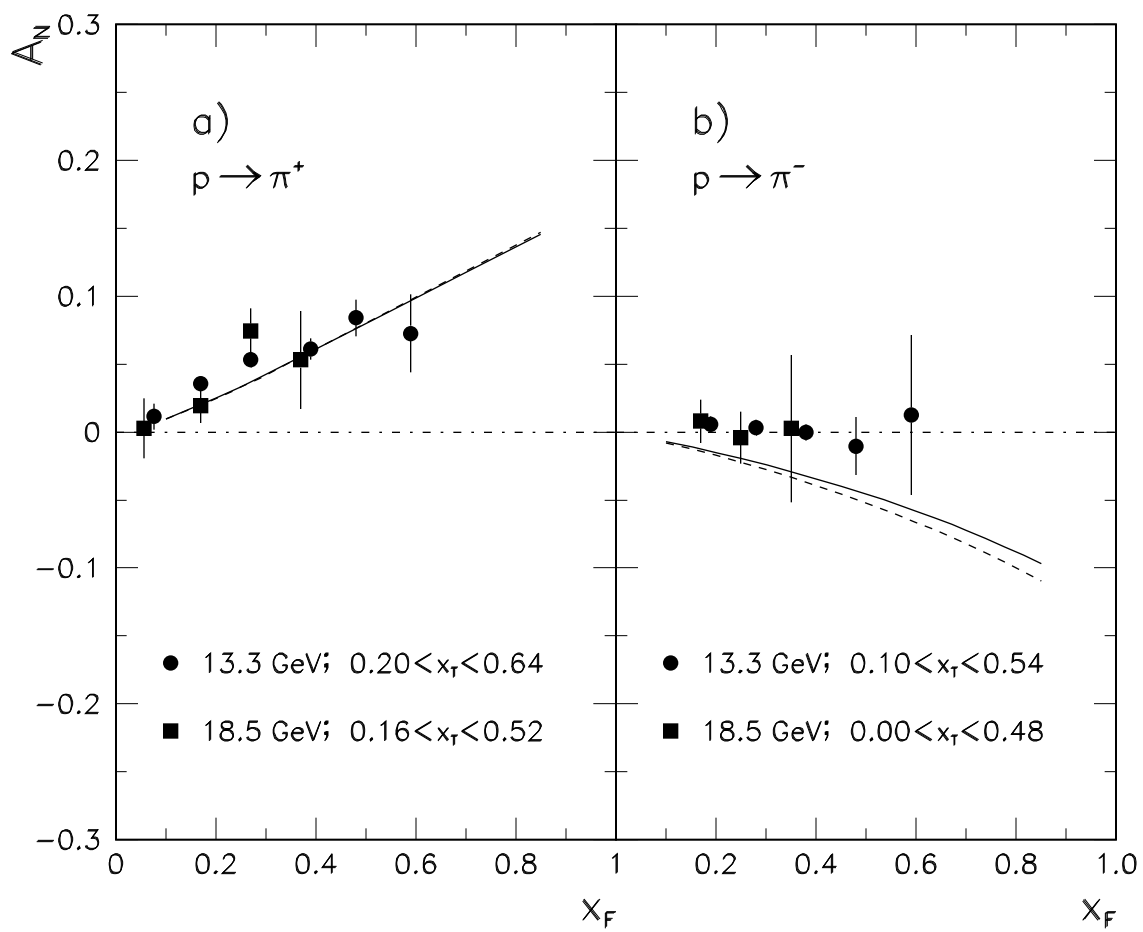


Fig.7

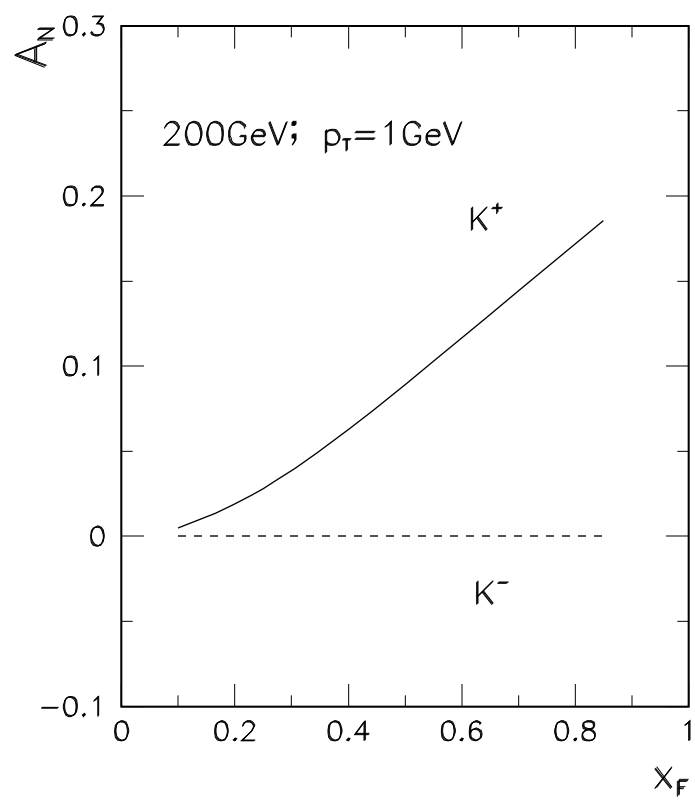


Fig.8